



Tapered beamforming for concentric ring arrays

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ABSTRACT

In this paper, some conventional filtering windows are modified and applied to uniform concentric circular antenna arrays (UCCA) for spatial smoothing and sidelobe reduction. The modified windows are applied to individual rings of the array that will taper the corresponding current amplitudes. The resulted sidelobe level, beamwidth and stability for amplitude errors are discussed for the different proposed tapering windows where it shows a sidelobe reduction to about 49 dB as in the case of Binomial UCCA while the Hamming window shows the most immunity to tapered amplitude errors.

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1. Introduction

Concentric circular antenna arrays has an interesting features over other array configurations such as linear one-dimensional or two-dimensional arrays [1–6]. It has widespread use in various applications such as mobile, radar, sonar and direction finding. The array consists of concentric rings each has a number of elements arranged in a circle of certain radius. The sidelobe level in this array is 17.5 dB for most sizes which is less than that of the two-dimensional arrays by 4 dB [5]. A popular array geometry is the uniform concentric circular arrays (UCCA) in which the rings as well as the individual ring elements are separated by almost half of the wavelength [6,7]. If the number of elements of the neighbored rings is incremented by 6 elements [7], the rings will be separated by the nearest distance to a half-wavelength (about 0.4775 of the wavelength). The UCCA at this separating distance will have the optimum radiation pattern and good predicted sidelobes locations but still has higher sidelobes levels which are not suitable in many applications requiring lower sidelobes. The problem of higher sidelobes can be solved through the tapered beamforming techniques in which the array feeding currents are tapered in amplitudes so that it has the maximum value at the center of the array and falls to the minimum at its ends. This technique is studied and performed for the one-dimensional linear arrays and some tapered beamforming techniques such as Binomial, Dolph-Chebyshev and others had proposed [5]. On the other hand, a similar technique which is equivalent to the tapered beamforming in filter design to improve the stopband characteristics (sidelobes) by windowing where some

windows such as Triangular, Hamming, Hanning, Blackman, Binomial and others are used [8]. Therefore, in this paper we will modify these filtering windows to be applied to the UCCA for radiation pattern smoothing and sidelobe reduction and the beamwidth variation of the mainlobe is depicted for the different windows. Also the array stability against tapered amplitude errors is discussed for the different tapering windows and the immunity against these errors is shown. The paper is arranged as follows: in Section 2, the array geometry of the UCCA and its related parameters are displayed. Section 3 introduces the different beamforming windows applied for tapering the UCCA and Section 4 discusses the sidelobe level and beamwidth variations. Section 5 discusses the stability of the array against amplitude errors and finally, Section 6 concludes the paper.

2. Uniform concentric circular arrays (UCCA)

Fig. 1 displays the geometry of a concentric circular antenna array consisting of M concentric rings each has a number of elements N_m where $m = 1, 2, \dots, M$. The elements in each ring are assumed to be omnidirectional and the interelement separation is almost half of the wavelength which can be obtained if the number of elements in the rings is incremented by 6 [7] or:

$$N_{m+1} = N_m + 6 \quad (1)$$

The separating distance of $\lambda/2$ is chosen to have a radiation pattern that has one mainlobe and no grating lobes which appear at larger separating distances. Also, the radiation pattern has wider beamwidth if we used a smaller interelement separation which reduces the efficiency of the array. If the mutual coupling between the neighbored elements is neglected, we can determine an expression for the array factor at any direction if we know the weights of the rings and the array steering matrix.

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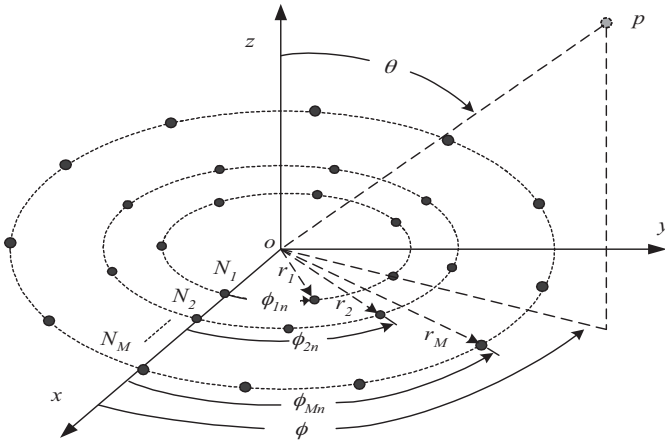


Fig. 1. Concentric circular arrays (CCA).

For the UCCA, the array steering matrix can be given by [7]:

$$AS(\theta, \phi) = [S_1(\theta, \phi)S_2(\theta, \phi), \dots, S_m(\theta, \phi), \dots, S_M(\theta, \phi)] \quad (2)$$

where each column in $AS(\theta, \phi)$ represents the ring steering vector which generally for the m th ring is given by:

$$S_m(\theta, \phi) = [e^{jkr_m \sin \theta \cos(\phi - \phi_{m1})}, e^{jkr_m \sin \theta \cos(\phi - \phi_{m2})}, \dots, e^{jkr_m \sin \theta \cos(\phi - \phi_{mN_m})}]^T \quad (3)$$

where $k = 2\pi/\lambda$ and this m th ring has a radius r_m and number of elements N_m .

In tapered beamforming, we multiply the array steering matrix with a tapering weight matrix $W(\theta, \phi)$ given by:

$$W(\theta, \phi) = [\alpha_1 S_1(\theta_o, \phi_o) \alpha_2 S_2(\theta_o, \phi_o), \dots, \alpha_m S_m(\theta_o, \phi_o), \dots, \alpha_M S_M(\theta_o, \phi_o)] \quad (4)$$

where for $m = 1, 2, \dots, M$, α_m is the amplitude coefficients of the m th ring current and $S_m(\theta_o, \phi_o)$ is the ring steering vector at the mainlobe direction (θ_o, ϕ_o) . From Eq. (4), we notice that all elements in an individual ring is weighted by the same value therefore the array factor will be given by

$$G(\theta, \phi) = \text{SUM}\{W(\theta, \phi)^H AS(\theta, \phi)\} \quad (5)$$

where the SUM operator is the summation of the elements of the resulted matrix and H is the complex conjugate transpose.

In this section, some conventional windows are modified and applied for amplitude tapering of the UCCA. These windows are well defined for filtering applications such as finite impulse response (FIR) filter designs such as Triangular, Hamming, Hanning, Blackman and Binomial windows. It had showed the possibility to reduce the sidelobe-to-mainlobe ratio in the filter magnitude response. Conventional one-dimensional tapered arrays have tapered the currents of the individual array elements, while in the case of UCCA, we consider the individual ring to be equivalent to an element of the one-dimensional linear array. The following sections defines these possible amplitude tapering windows.

3. Tapering windows for UCCA

In this section, some conventional windows are modified and applied for amplitude tapering of the UCCA. These windows are well defined for filtering applications such as finite impulse response (FIR) filter designs such as Triangular, Hamming, Hanning, Blackman and Binomial windows. It had showed the possibility to reduce the sidelobe-to-mainlobe ratio in the filter magnitude response. Conventional one-dimensional tapered arrays have tapered the currents of the individual array elements, while in the case of UCCA,

we consider the individual ring to be equivalent to an element of the one-dimensional linear array. The following sections defines these possible amplitude tapering windows.

3.1. Uniform feeding window

The uniformly fed UCCA has the same amplitude coefficients which is the unity or

$$\alpha_m = 1, \quad m = 1, 2, \dots, M \quad (6)$$

these coefficients give the smallest beamwidth compared with any other window and the highest sidelobe level of 17.5 dB as shown in Fig. 2a for a typical array of $N_1 = 5$ and $M = 10$.

3.2. Triangular amplitude tapering

In Triangular tapering, the amplitude weighting follows a triangular function that equals zero at a virtual ring number $M + 1$. The rings amplitude coefficients for this scheme is given by:

$$\alpha_m = \frac{(M - m + 1)}{M}, \quad m = 1, 2, \dots, M \quad (7)$$

where m is the ring number in the array. The innermost ring has a weight value $\alpha_1 = 1$ while the outermost ring has a weight value of $\alpha_M = 1/M$.

Fig. 2b displays a typical radiation pattern of the same array configuration as in Fig. 2a.

3.3. Hamming amplitude tapering

The Hamming window [8] used for filter applications are modified here and gives the following rings coefficients for a UCCA of M rings:

$$\alpha_m = 0.54 - 0.46 \cos\left(\frac{\pi(m - M - 2)}{M + 1}\right), \quad m = 1, 2, \dots, M \quad (8)$$

Fig. 2c displays the radiation pattern of Hamming UCCA where the sidelobe level will be 29.5 dB.

3.4. Hanning amplitude tapering

The Hanning window [8] is very similar to the Hamming window and provides an array coefficients that are modified to suit the application of the UCCA and is given by:

$$\alpha_m = 0.5 - 0.5 \cos\left(\frac{\pi(m - M - 2)}{M + 1}\right), \quad m = 1, 2, \dots, M \quad (9)$$

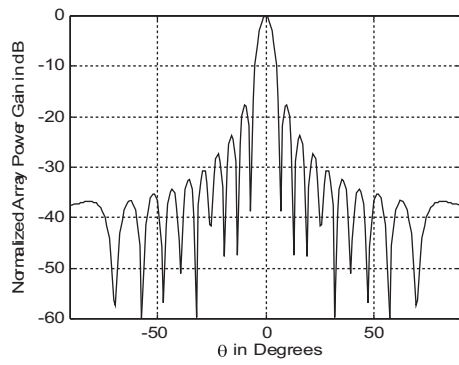
Fig. 2d depicts the radiation pattern of the Hanning tapered array which has a very similar value of the sidelobe level as in the Hamming array.

3.5. Blackman amplitude tapering

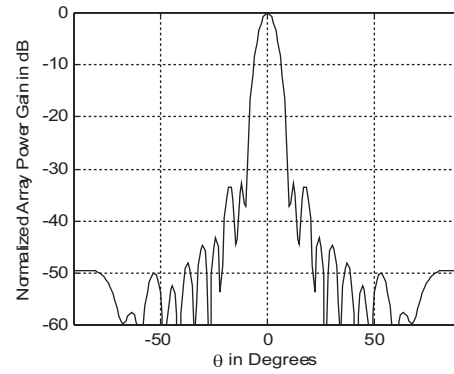
Invariant to the last tapering schemes, Blackman window [8] provides another cosine term for further sidelobe reduction. The modified coefficients function for the UCCA tapering is given by:

$$\alpha_m = 0.42 - 0.5 \cos\left(\frac{\pi(m - M - 2)}{M + 1}\right) + 0.08 \cos\left(\frac{2\pi(m - M - 2)}{M + 1}\right), \quad m = 1, 2, \dots, M \quad (10)$$

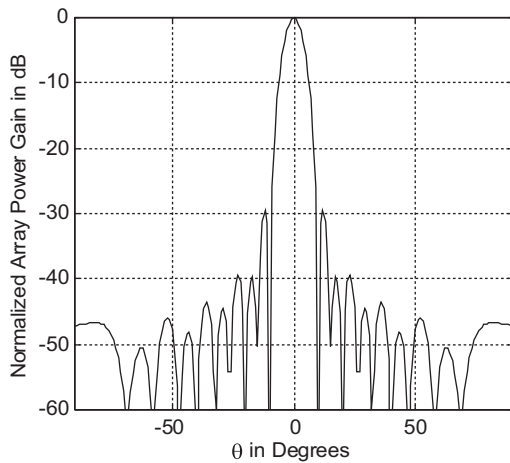
The radiation pattern for this type of tapering is shown in Fig. 2e which reduces the sidelobe level to 38 dB.



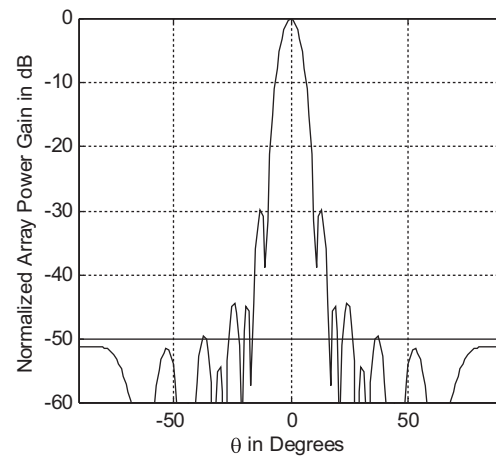
a. Typical radiation pattern of uniformly fed UCCA of $N_1 = 5$ and $M = 10$.



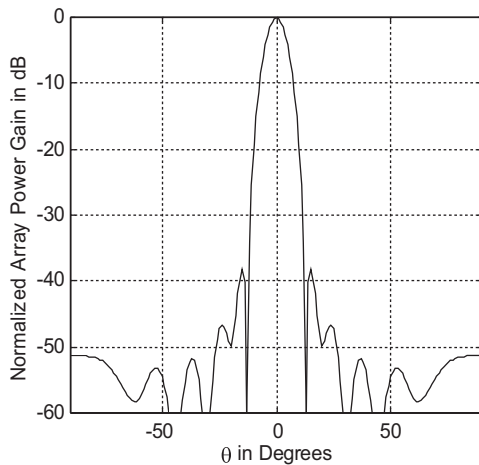
b. Typical radiation pattern of Triangular UCCA of $N_1 = 5$ and $M = 10$.



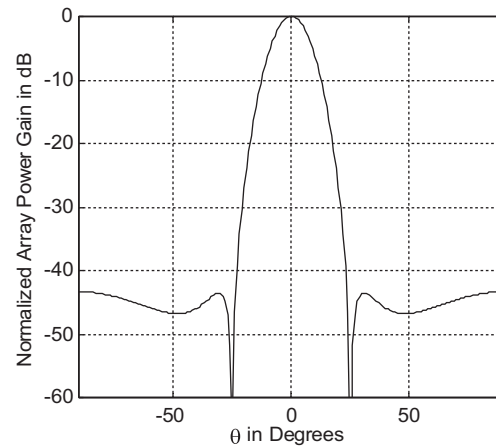
c. Typical radiation pattern of Hamming UCCA of $N_1 = 5$ and $M = 10$.



d. Typical radiation pattern of Hanning UCCA of $N_1 = 5$ and $M = 10$.



e. Typical radiation pattern of Blackman UCCA of $N_1 = 5$ and $M = 10$.



f. Typical radiation pattern of Binomial UCCA of $N_1 = 5$ and $M = 10$.

Fig. 2. (a) Typical radiation pattern of uniformly fed UCCA of $N_1 = 5$ and $M = 10$. (b) Typical radiation pattern of Triangular UCCA of $N_1 = 5$ and $M = 10$. (c) Typical radiation pattern of Hamming UCCA of $N_1 = 5$ and $M = 10$. (d) Typical radiation pattern of Hanning UCCA of $N_1 = 5$ and $M = 10$. (e) Typical radiation pattern of Blackman UCCA of $N_1 = 5$ and $M = 10$. (f) Typical radiation pattern of Binomial UCCA of $N_1 = 5$ and $M = 10$.

Table 1
Binomial coefficients of the UCCA.

M	k	Binomial coefficients for linear arrays of k elements	Binomial coefficients for UCCA
1	2	1, 1	1
2	4	1, 3, 3, 1	3, 1
3	6	1, 5, 10, 10, 5, 1	10, 5, 1
4	8	1, 7, 21, 35, 35, 21, 7, 1	35, 21, 7, 1
5	10	1, 9, 36, 84, 126, 126, 84, 36, 9, 1	126, 84, 36, 9, 1

3.6. Binomial amplitude tapering

The Binomial amplitude feeding [5] of the UCCA can be obtained if we apply the Binomial coefficients to the ring arrays. Starting with the Binomial expansion for the following expression:

$$(1+x)^{k-1} = 1 + (k-1)x + \frac{(k-1)(k-2)}{2!}x^2 + \frac{(k-1)(k-2)(k-3)}{3!}x^3 + \dots \quad (11)$$

where for linear one-dimensional Binomial arrays, the elements coefficients are taken as the first k positive coefficients. In the UCCA case, we assume that k=2M and find the first k coefficients in Eq. (11), then the binomial amplitude window is taken as the values of these coefficients from M+1 to 2M as depicted in Table 1.

For example, if we have M=4, then the innermost ring will be weighted in amplitude by α₁=35 and for the second outer ring α₂=21, etc., while for any number of rings in the array, the outermost ring will have α_M=1. Binomial window will provide the lowest possible sidelobe level and the largest beamwidth compared with any other tapering scheme. A typical radiation pattern of Binomial UCCA of the same configuration as in the last cases is depicted in Fig. 2f where the sidelobe level will be 43.5 dB.

Fig. 3 displays the variations of the different proposed windows for a UCCA of N₁=5 and M=10 where the values are normalized with respect to that of the innermost ring. In this figure, the weights of the Binomial window has the largest values spread which introduces some practical limitations and will be sensitive to amplitude errors while the lowest spread in values occurs in the case of Hamming window.

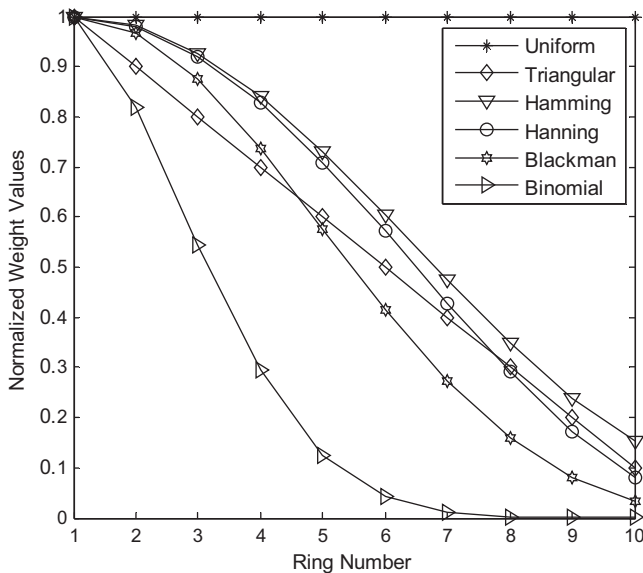


Fig. 3. Typical values of the windows for tapered UCCA of M=10.

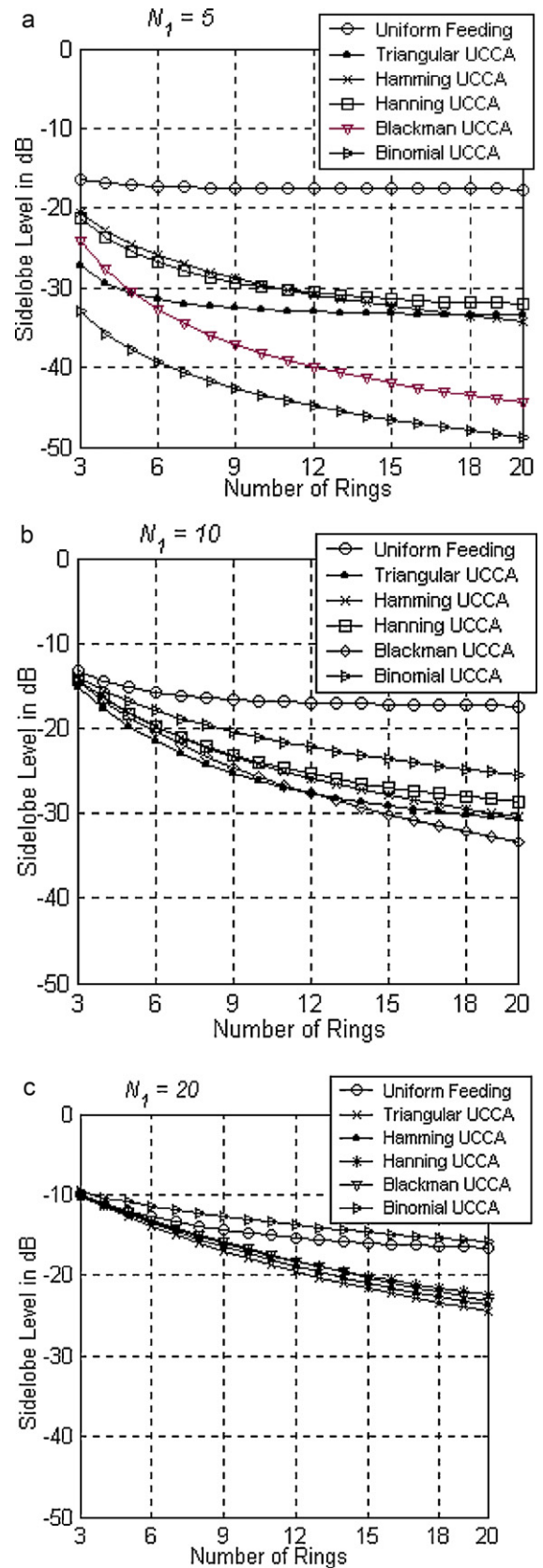


Fig. 4. (a) Sidelobe level variation of tapered UCCA at N₁=5. (b) Sidelobe level variation of tapered UCCA at N₁=10. (c) Sidelobe level variation of tapered UCCA at N₁=20.

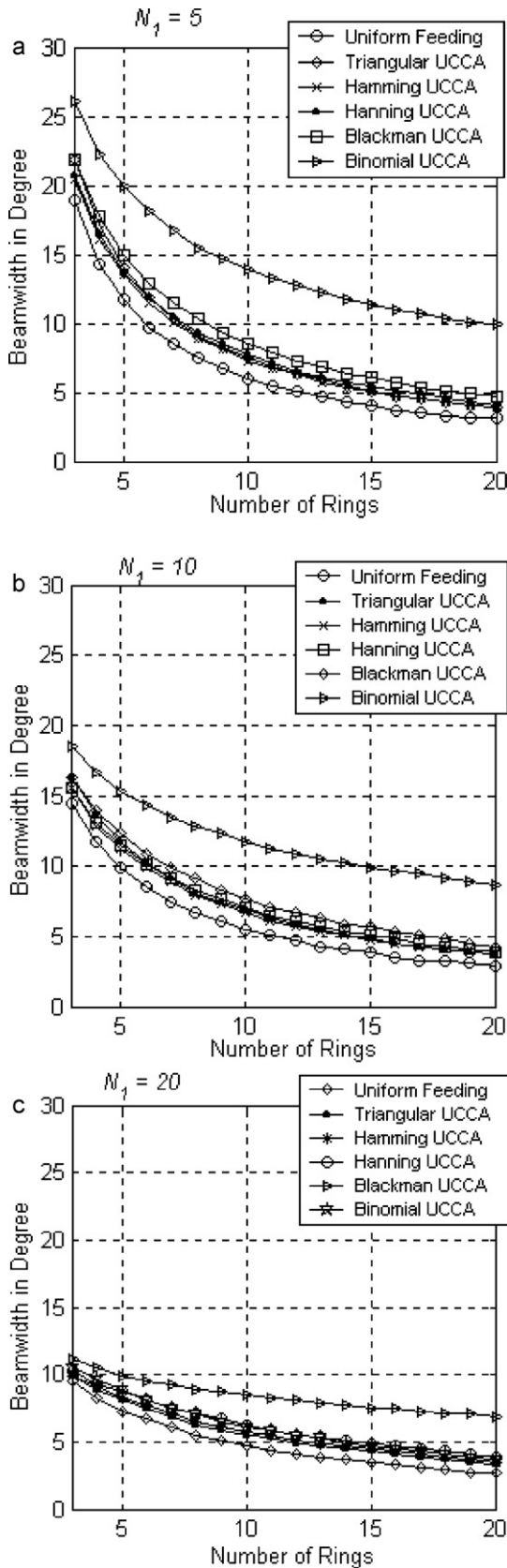


Fig. 5. (a) Beamwidth variation of tapered UCCA at $N_1 = 5$. (b) Beamwidth variation of tapered UCCA at $N_1 = 10$. (c) Beamwidth variation of tapered UCCA at $N_1 = 20$.

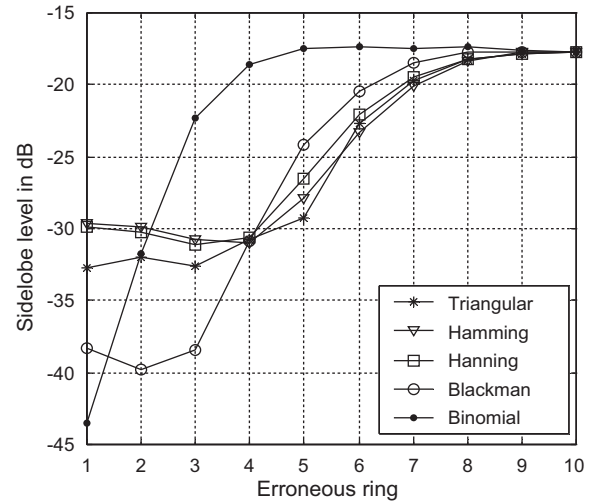


Fig. 6. Sidelobe degradation as a function of the error location within the UCCA.

4. Sidelobe and beamwidth performance

The maximum sidelobe level and the mainlobe beamwidth are discussed in this section for the different weighting schemes. Fig. 4a–c displays the sidelobe levels in dB for the tapered UCCA as a function of the number of rings at different internal ring size. From these figures we notice in general that the sidelobe level will decrease for all windows with increasing the number of rings in the array at a specific innermost size. The uniform feeding case provides the highest sidelobe level, while the Binomial tapering provides the lowest possible levels especially for lower number of elements in the innermost ring and approaches 49 dB below the mainlobe at $M = 20$ ring for the UCCA of $N_1 = 5$ as shown in Fig. 4a. If we increase the number of elements in the innermost ring, the tapered UCCA performance degrades for all windows and the sidelobes will raise again as depicted in Fig. 4b and c. Also the curves in these figures will converge more and become closer together at larger number of elements in the innermost ring.

On the other hand, Fig. 5a–c depict the beamwidth of the mainlobe for the different feeding schemes where it decreases with both increasing the number of rings in the array and the number of elements of the innermost ring. The Binomial window will result in the largest beamwidth while in all cases the lowest beamwidth is obtained from the uniform feeding. As noticed with the sidelobe performance, the curves of the beamwidth will converge more together as the number of elements in the innermost ring increases.

5. Stability of tapered UCCA

A very important issue of the tapered beamforming is the stability of the array performance against the amplitude feeding errors. These errors will affect greatly the radiation pattern of the UCCA and result in higher sidelobes. The performance of the UCCA with tapering errors can be discussed assuming an erroneous ring feeding and show the effect on the sidelobe level. These errors may occur at any ring and it is expected that the errors in the outer rings will result in more degradation than if it was in the inner ones. As a case study, we consider a tapered UCCA of $N_1 = 5$ and $M = 10$ and assuming an error that can be occurred in any ring results from changing the coefficient value to be equal to 1. Fig. 6 depicts the sidelobe level of this tapered UCCA as a function of the error location for various proposed tapering windows where the sidelobe level for all windows will converge to 17.5 dB when the error occurs in the outermost ring. The increase in the sidelobe level is faster in the case of Binomial tapering while there is an immunity showed

in the case of Hamming window. To clarify the amount of degradation in the array performance, we take the difference between the resulted sidelobe levels before and after the feeding errors as shown in Fig. 6. In this figure, the difference is largest for the Binomial window while the Hamming window shows the smallest difference indicating an immunity to amplitude errors compared with the other schemes.

6. Conclusion

In this paper, some conventional windows are modified to suit the tapered beamforming for the UCCA. These windows are discussed in details and the array performance in terms of the sidelobe levels and the beamwidth variations are discussed. It has been noticed that the sidelobe level and beamwidth for any window are sensitive to the number of elements in the innermost ring and the number of rings in the array. Also the lowest possible sidelobe level occurs in the case of Binomial tapering while it provides the maximum beamwidth compared with the other windows. The array performance with the tapered amplitude errors is also discussed and showed that the Binomial windowing is highly sensitive with

amplitude variations while the Hamming window has the highest immunity against these errors.

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